

6.4 p1

MA121

June 18/8

The substitution Method

From the chain rule, $\frac{d}{dx} (f(g(x))) = f'(g(x)) g'(x)$,
 the antiderivative is $\int f'(g(x)) g'(x) dx = f(g(x)) + C$

OR we substitute $u = g(x)$, then we have

$$\int f'(g(x)) g'(x) dx = \int f'(u) \frac{du}{dx} dx = \int f'(u) du$$

$$= f(u) + C = f(g(x)) + C$$

2x 2 p 198

$$\int (2x^2 - 3)^5 4x dx, \text{ let } u = 2x^2 - 3 \Rightarrow \frac{du}{dx} = 4x \Rightarrow du = 4x dx$$

$$\text{we have } \int (2x^2 - 3)^5 4x dx = \int u^5 \frac{du}{dx} dx = \int u^5 du$$

$$= \frac{1}{6} u^6 + C = \frac{1}{6} (2x^2 - 3)^6 + C$$

Check:

$$\frac{d}{dx} \left[\frac{1}{6} (2x^2 - 3)^6 + C \right] = \frac{1}{6} [6 (2x^2 - 3)^5 (2x^2 - 3)']$$

$$= (2x^2 - 3)^5 4x$$

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$$\int 2(3x+1)^6 dx, \text{ let } u = 3x+1 \Rightarrow \frac{du}{dx} = 3 \Rightarrow du = 3 dx$$

$$\Rightarrow \frac{1}{3} du = dx, \text{ so we have } \int 2(3x+1)^6 dx$$

$$= \int 2u^6 \left(\frac{1}{3}\right) du = \frac{2}{3} \int u^6 du = \frac{2}{3} \left[\frac{1}{7} u^7 \right] + C$$

$$= \frac{2}{21} u^7 + C = \frac{2}{21} (3x+1)^7 + C$$

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$$\int (3-x^4)^3 x^3 dx, \text{ let } u = 3-x^4 \Rightarrow du = -4x^3 dx$$

$$= -\frac{1}{4} du = x^3 du, \text{ so we have } \int (3-x^4)^3 x^3 dx$$

$$= \int u^3 \left(-\frac{1}{4} du\right) = -\frac{1}{4} \int u^3 du = -\frac{1}{4} \left[\frac{1}{4} u^4 \right] + C$$

$$= -\frac{1}{16} (3-x^4)^4 + C$$

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$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx, \text{ let } u = e^x + e^{-x} \Rightarrow \frac{du}{dx} = e^x + e^{-x}(-1) = e^x - e^{-x}$$

$$\Rightarrow du = (e^x - e^{-x}) dx \text{ so we have } \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$= \int \frac{(e^x - e^{-x}) dx}{e^x + e^{-x}} = \int \frac{du}{u} = \ln |u| + C = \ln(e^x + e^{-x}) + C$$

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$$\int \frac{(\ln x)^{1/2}}{x} dx, \text{ let } u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$\text{so we have } \int \frac{(\ln x)^{1/2}}{x} dx = \int (\ln x)^{1/2} \frac{1}{x} dx = \int u^{1/2} du$$

$$= \frac{u^{3/2}}{3/2} + C = \frac{2}{3} u^{3/2} + C = \frac{2}{3} (\ln x)^{3/2} + C$$

Solving Definite Integrals using the Method of Substitution

We need to rewrite the limits of the definite integral based on our substitution.

If we let $u = g(x)$, then we have

$$\int_{x=a}^{x=b} f'(g(x)) g'(x) dx = \int_{u=g(a)}^{u=g(b)} f'(u) du$$

ex 20 p 198

$$\int_3^5 \frac{x}{5-x^2} dx, \text{ let } u = 5-x^2 \Rightarrow du = -2x dx \Rightarrow -\frac{1}{2} du = x dx$$

$$\text{when } x=5, \text{ then } u = 5-5^2 = -20$$

$$\text{when } x=3, \text{ then } u = 5-3^2 = -4, \text{ so we have}$$

$$\int_3^5 \frac{x dx}{5-x^2} = \int_{-4}^{-20} \frac{-\frac{1}{2} du}{u} = -\frac{1}{2} [\ln |u|]_{-4}^{-20}$$

$$= -\frac{1}{2} (\ln 20 - \ln 4) = -\frac{1}{2} (\ln \frac{20}{4}) = -\frac{1}{2} \ln 5.$$

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$$\int_0^2 2x e^{x^2} dx, \text{ let } u = e^{x^2} \Rightarrow du = e^{x^2} (2x) dx \Rightarrow du = 2x e^{x^2} dx$$

$$\text{when } x=2, \text{ then } u = e^{2^2} = e^4$$

$$\text{when } x=0, \text{ then } u = e^{0^2} = e^0 = 1 \text{ so we have}$$

$$\int_0^2 2x e^{x^2} dx = \int_1^{e^4} du = \int_1^{e^4} 1 du = [u]_1^{e^4} = e^4 - 1$$

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Given $MR(q) = \frac{30q + 7000}{q + 100}$, we want to calculate the additional revenue when we increase production from 30 units to 50 units. We want:

$$\int_{30}^{50} MR(q) dq = \int_{30}^{50} \frac{30q + 7000}{q + 100} dq = \int_{30}^{50} \left(30 + \frac{4000}{q + 100} \right) dq$$

$$= \left[30q + 4000 \ln|q + 100| \right]_{30}^{50}$$

$$= [30(50) + 4000 \ln(50 + 100)] - [30(30) + 4000 \ln(30 + 100)]$$

$$= 1500 + 4000 \ln 150 - 900 - 4000 \ln 130$$

$$= 600 + 4000 (\ln 150 - \ln 130) = 600 + 4000 \ln \frac{150}{130}$$

$$= 600 + 4000 \ln \frac{15}{13}$$

or let $u = q + 100 \Rightarrow du = dq$. Also from $u = q + 100 \Rightarrow q = u - 100$

When $q = 50$, then $u = 50 + 100 = 150$

When $q = 30$, then $u = 30 + 100 = 130$, so we have

$$\int_{30}^{50} \frac{30q + 7000}{q + 100} dq = \int_{130}^{150} \frac{3(u - 100) + 7000}{u} du = \int_{130}^{150} \frac{30u - 3000 + 7000}{u} du$$

$$= \int_{130}^{150} \frac{30u + 4000}{u} du = \int_{130}^{150} \left(\frac{30u}{u} + \frac{4000}{u} \right) du = \int_{130}^{150} \left(30 + \frac{4000}{u} \right) du$$

$$= \left[30u + 4000 \ln|u| \right]_{130}^{150}$$

$$= [30(150) + 4000 \ln 150] - [30(130) + 4000 \ln 130]$$

$$= 4500 + 4000 \ln 150 - 3900 - 4000 \ln 130$$

$$= 600 + 4000 (\ln 150 - \ln 130) = 600 + 4000 \ln \frac{150}{130}$$

$$= 600 + 4000 \ln \frac{15}{13}$$